Midterm / 2022.10.6 / MAT 4233.001 / Modern Abstract Algebra

Name: ____________________

Please show all work.

1. Determine for which natural numbers we have $n! > 2^n$ and prove it by induction.

2. Suppose $\alpha, \beta \in \Sigma_n$ are permutations of $\{1,2,3,4,5\}$ given by

$$\begin{array}{c|ccccc} x & 1 & 2 & 3 & 4 & 5 \\ \alpha(x) & 3 & 2 & 4 & 1 & 5 \end{array} \qquad \begin{array}{c|ccccc} x & 1 & 2 & 3 & 4 & 5 \\ \beta(x) & 4 & 5 & 1 & 3 & 2 \end{array}$$

   Find $(\alpha\beta)^{-1}$ and $\alpha^{-1}\beta^{-1}$

3. Suppose G is a finite group and $x \in G$. Prove:

   (a) x has finite order.

   (b) $x^n = e$ if and only if the order of x divides n.

4. Prove that the set of all complex fourth roots of unity $H = \left\{z \in \mathbf{C} \colon z^4 = 1\right\}$ is a cyclic subgroup of $\mathbf{C}^*$ of order 4

1	2	3	4	total (40)