Name: $\qquad$

Please show all work.

1. Suppose $m$ and $n$ are natural numbers. Prove that
(a) any common divisor of $m$ and $n$ divides $\operatorname{gcd}(m, n)$
(b) $\operatorname{lcm}(m, n)$ divides any common multiple of $m$ and $n$
2. Use the extended Euclid's algorithm to find the multiplicative inverse of 17 modulo 37
3. Determine for which natural numbers $n$ we have $n!>2^{n}$ and prove it by induction.
4. Prove that $\{1,-1\} \subseteq \mathbf{Z}$ is a multiplicative group and that it is isomorphic to $\mathbf{Z}_{2}$

| 1 | 2 | 3 | 4 | total (40) |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |

