1. Let $G = \{x \in \mathbf{R}\colon (\exists m, n \in \mathbf{Z})\ x = m + n\sqrt{2}\}$.

   (a) Prove m, n are unique for each $x \in G$.
   Hints: (i) subtract, (ii) prove $m + n\sqrt{2} = 0 \Rightarrow m = n = 0$

   (b) Prove G is a group under the usual addition of real numbers.

2. Let G be as above.

   (a) Prove the relation on $\mathbf{R}$ defined by $x \sim x' \Leftrightarrow x - x' \in G$ is an equivalence relation.

   (b) Describe the equivalence class of 0 with respect to the above equivalence relation. List several typical representatives of the class. Same for the equivalence class of π.

   (c) Show that addition of equivalence classes is well defined.
   Hint: prove $x \sim x', y \sim y' \Rightarrow x + y \sim x' + y'$

3. Suppose G is a group and for each $s \in G$ define $\varphi_s(x) = sxs^{-1}$

   (a) Show that $(\forall s \in G)\ \varphi_s$ is an automorphism of G

   (b) Prove G is Abelian $\Leftrightarrow (\forall s \in G)\ \varphi_s$ is the identity on G