

1. Let $\alpha = (1, 7, 6)(2, 5)(3, 4, 8, 9) \in S_9$ (permutation in cycle notation)
 - (a) What is the order of α ? Explain.
 - (b) What is the parity of α ? Explain.
 - (c) Express α^{2027} as a product of disjoint cycles. Explain.
2. In each case, exhibit, with proof, a concrete example of a nontrivial proper subgroup H of the symmetric group S_3 such that
 - (a) H is normal in S_3
 - (b) H is not normal in S_3
3. Use the Chinese remainder formula to find all solutions to the system of congruences:
$$x \equiv 2 \pmod{5} \quad x \equiv 2 \pmod{7} \quad x \equiv 5 \pmod{8}$$
4.
 - (a) How many group homomorphisms $\varphi: \mathbf{Z} \rightarrow \mathbf{Z}_{54}$ are there?
 - (b) How many of these φ are injective and how many are surjective?
 - (c) Prove your assertions.