Midterm 2 / 2019.11.26 / MAT 4233.001 / Modern Abstract Algebra

Name: _____

Please show all work. If you use a theorem, name it or state it.

1. Let $\alpha = (3, 5, 1)(4, 2, 1, 3)$ be a permutation (in cycle notation). Express α as a product of disjoint cycles. What are the order and the parity of α? Explain. Simplify α^{11}.

2. Prove that the set of all rotations in the dihedral group D_n is a normal subgroup. What can you say about the quotient group?

3. Suppose $\varphi\colon \mathbf{Z}_{15} \to \mathbf{Z}_3 \oplus \mathbf{Z}_5$ is a group isomorphism. If $\varphi(2) = [2, 3]$, what is $\varphi(1)$?

4. Suppose S is a ring with p elements, where p is prime.

   (a) Show that as an additive group (ignoring multiplication for the moment), S is cyclic.
   Hint: Consider the subgroup generated by a nonzero element of S.

   (b) Show that S is a commutative ring.
   Hint: Use part (a).

1	2	3	4	total (40)