Midterm 2 / 2019.4.25 / MAT 4233.001 / Modern Abstract Algebra

Name: ____

Please show all work. If you use a theorem, name it or state it.

1. Let $\alpha = (3, 4, 1)(5, 2, 1, 3)$ be a permutation (in cycle notation). Express α as a product of disjoint cycles. What are the order and the parity of α? Explain. Simplify α^{2019}.

2. Prove that any group of prime order is cyclic.

3. Suppose $m, n, k \in \mathbf{N}$ with $\operatorname{lcm}(m, n) = k$. Define a group homomorphism $\varphi\colon \mathbf{Z} \to \mathbf{Z}_m \oplus \mathbf{Z}_n$ by $\varphi(i) = [i \bmod m, i \bmod n]$. Prove that $\ker \varphi = k\mathbf{Z}$. What does the first isomorphism theorem tell you about the image of φ? What can you say about $\mathbf{Z}_m \oplus \mathbf{Z}_n$ if $\gcd(m, n) = 1$?

4. Let F be a field. Show that the set of all polynomials in $F[x]$ with zero constant term is a maximal ideal. What is the quotient ring?

1	2	3	4	total (40)