Midterm 1 / 2019.2.27 / MAT 4233.001 / Modern Abstract Algebra

Name: ____

Please show all work. If you use a theorem, name it or state it.

1. Suppose m and n are natural numbers. Prove that
   (a) any common divisor of m and n divides $\gcd(m, n)$.
   (b) $\operatorname{lcm}(m, n)$ divides any common multiple of m and n.
2. Sketch the subgroup lattice for $\mathbf{Z}_{20}$. For each subgroup, list all the elements and indicate all possible generators of the subgroup.
3. Suppose an element x of the dihedral group D_n is a composition (in an arbitrary order) of j rotations and k reflections (flips). [Example: $x = r_3 f_2 r_1 r_2 f_1$ with $j = 3$ and $k = 2$] Under what conditions on j and k is x a rotation? A reflection? Explain.
4. Suppose G is a finite group and $x \in G$. Prove:
   (a) x has finite order.
   (b) $x^n = e$ if and only if the order of x divides n.
5. Let $\mathbf{R}^+$ denote the multiplicative group of positive real numbers. Suppose $a \in \mathbf{R}, a > 1$. Prove that the exponential map $x \mapsto a^x$ is an isomorphism from $\mathbf{R}$ to $\mathbf{R}^+$.

1	2	3	4	5	total (50)