Midterm 2 / 2018.4.19 / MAT 4233.001 / Modern Abstract Algebra

Name: ______________________________

Please show all work. If you use a theorem, name it or state it.

1. Prove that the special linear group $\mathrm{SL}_n(\mathbf{R})$ of all matrices with determinant 1 is a normal subgroup of the general linear group $\mathrm{GL}_n(\mathbf{R})$ of all invertible $n \times n$ matrices with real coefficients. What is the quotient group?

2. Prove that the set of all rotations in the dihedral group D_n of all symmetries of the regular polygon with n vertices is a normal subgroup. What is the quotient group?

3. Suppose X is a set and F is a field. Let R be the ring of all functions $X \to F$ with pointwise operations.

   (a) What are the units of R? Prove your assertion.

   (b) Use an explicit example to show that R may have zero divisors.

4. With R as in the preceding problem and $s \in X$, let $I = \{f \in R\colon f(s) = 0\}$. Prove that I is a maximal ideal of R.

1	2	3	4	total (40)