Midterm 1 / 2018.2.22 / MAT 4233.001 / Modern Abstract Algebra

Name: ____________________

Please show all work. If you use a theorem, name it or state it.

1. Suppose m and n are natural numbers. Prove that
   (a) any common divisor of m and n divides $\gcd(m, n)$.
   (b) $\operatorname{lcm}(m, n)$ divides any common multiple of m and n.
2. Let $\alpha = (1, 2, 5, 4)(2, 6, 3)(5, 6, 3, 2, 1)$ be a permutation (in cycle notation). Express α as a product of disjoint cycles. What is the order of α? Simplify α^{61}.
3. Suppose G is a group and every element, other than the identity, has order 2. Prove G is commutative.
4. Suppose G is a multiplicative group, $x \in G$ and n is a natural number. Prove that $x^n = e$ if and only if the order of x divides n.
5. Define $\varphi, \psi : \mathbf{C}^* \to \mathbf{C}^*$ by $\varphi(z) = z^5$ and $\psi(z) = |z|$. Prove that φ and ψ are group homomorphisms. Describe and sketch their kernels. Are they cyclic groups? Explain.

1	2	3	4	5	total (50)