Midterm 2 / 2013.4.24 / MAT 4233.001 / Modern abstract algebra

Name: ______________________________

Please show all work and justify your answers.

1. Let $G = \mathbf{Z}_4 \oplus \mathbf{Z}_4$, $H = \langle [2,0],[0,2] \rangle$, and $K = \langle [1,2] \rangle$. Determine the isomorphism classes of G/H and G/K as finite abelian groups, i.e. represent each group as an external product of $\mathbf{Z}_n$'s.

2. Suppose G is a finite group, $H \triangleleft G$ and $K \triangleleft G$, Prove that $HK \triangleleft G$.

   Hint: don't forget to show $HK < G$ first.

3. Suppose φ is a non-injective group homomorphism from $\mathbf{Z}_{11}$. Prove that φ is constant (which constant?).

4. Suppose R is a ring, $n > 1$ is a natural number, and $x^n = x$ for all $x \in R$. Prove that if $a, b \in R$ and $ab = 0$, then $ba = 0$.

5. Suppose R is a commutative ring with 500 elements, one of them unity, and I is an ideal of R with 100 elements. Prove that R/I is a field.

1	2	3	4	5	total (50)	%

Prelim. course grade: %