Midterm 1 / 2013.3.6 / MAT 4233.001 / Modern abstract algebra

Name: ____

Please show all work and justify your answers.

1. Show $\begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix} \in GL(2, \mathbf{Z}_{11})$ and find its inverse.
2. Determine the subgroup lattice of the symmetric group S_3. Pick a nontrivial subgroup of S_3 other than A_3 and find all its left cosets and right cosets.
3. Suppose G is an abelian multiplicative group with $a, b \in G$. Prove by induction that $(ab)^n = a^n b^n$ for all $n \in \mathbf{Z}$. Find an example to show that the result may not hold if G is not abelian.
4. Suppose a subgroup H of $\mathbf{Z}$ contains two distinct primes. Prove that $H = \mathbf{Z}$.
5. Suppose G is a multiplicative group and $a \in G$. Define $\varphi\colon \mathbf{Z} \to \langle a \rangle$ by $\varphi(k) = a^k$. Prove that φ is an isomorphism if and only if $|a| = \infty$.

1	2	3	4	5	total (50)	%

Prelim. course grade: %