Midterm 2 / 2011.4.25 / MAT 4233.001 / Modern abstract algebra

Name: ______________________________

Please show all work and justify your answers.

1. What hypotheses on m and n are needed to ensure that $\mathbf{Z}_{mn} \cong \mathbf{Z}_m \oplus \mathbf{Z}_n$? Show by example that if the hypotheses are not satisfied, then the conclusion fails to hold. Explain why your example works.
2. Exhibit a nontrivial proper subgroup of the symmetric group S_n that is normal. Same for not normal. Prove your assertions.
3. Let R be the ring of continuous functions $\mathbf{R} \to \mathbf{R}$ with the usual pointwise subtraction and multiplication. Which elements of R are units? Are there nonzero zero divisors in R? Let $A = \{f \in R \colon f(0) = 0\}$. Prove that A is an ideal of R. Is A a prime ideal? Maximal? Prove your assertions.
4. Prove that $x^2 + 1$ is an irreducible polynomial in $\mathbf{R}[x]$. Prove that the factor ring $\mathbf{R}[x]/\langle x^2 + 1\rangle$ is a field.

1	2	3	4	total (40)	%

Prelim. course grade: %