

Name: \_\_\_\_\_

Please show all work and justify your answers.

1. Find the isomorphism class of  $U(12)$  as a finite abelian group.
2. Find all ideals of  $\mathbf{Z}_{60}$ . Explain why that's all of them. Draw a lattice (i.e. sketch subset relations among the ideals).
3. Prove that  $\{\sigma \in S_3: \sigma(3) = 3\}$  is a subgroup of  $S_3$ . Is it abelian? Is it a normal subgroup of  $S_3$ ? Prove your assertions.
4. Find the quotient and remainder of  $x^4 + 3x^3 + 2x^2 + x - 1$  divided by  $2x^2 + 1$  in  $\mathbf{Z}_7[x]$ .
5. Let  $A = \{p \in \mathbf{R}[x]: p(0) = 0\}$ . Prove that  $A$  is an ideal of  $\mathbf{R}[x]$ . Is  $A$  a prime ideal? Maximal? Explain.

1	2	3	4	5	total (50)	%

Prelim. course grade: %