

Name: \_\_\_\_\_

Please show all work and justify your answers.

1. Prove that  $A_n \triangleleft S_n$  and that  $S_n/A_n \cong \mathbf{Z}_2$ .
2. Let  $A$  be the multiplicative group  $\{1, 9, 16, 22, 29, 53, 74, 79, 81\} \subset \mathbf{Z}_{91}$ . Find the isomorphism class of  $A$  as a finite abelian group.
3. Find all ideals of  $\mathbf{Z}_{18}$ . Draw a lattice.
4. Let  $A = \{p \in \mathbf{Z}[x]: p(0) = 0\}$ . Prove that  $A$  is an ideal of  $\mathbf{Z}[x]$ . Prove that  $A$  is a prime ideal, but not maximal.
5. Let  $A = \{(x, y) \in \mathbf{Z} \oplus \mathbf{Z}: y \text{ is even}\}$ . Prove that  $A$  is an ideal of  $\mathbf{Z} \oplus \mathbf{Z}$ . Prove that  $A$  is maximal.

1	2	3	4	5	total (50)	%

Prelim. course grade: %