Midterm 2 / 2008.4.24 / Modern abstract algebra / MAT 4233.001

Name: _____

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Let $H = \{(), (12)(34), (13)(24), (14)(23)\}$. Prove that H is a subgroup of A_4. What is its index $[A_4 : H]$? Is H is normal in A_4? Prove your assertion.

2. Let A be the set of all polynomials in $\mathbf{Z}[x]$ such that the constant coefficient is divisible by 3. Prove that A is an ideal of $\mathbf{Z}[x]$. Is it maximal? Prove your assertion.

3. Let R be the ring of continuous functions $\mathbf{R} \to \mathbf{R}$ with the usual pointwise operations. Is there a function in R that is neither a zero divisor nor a unit of R? Provide an explicit example or prove that no such example exists.

4. Suppose R is an integral domain. What is the largest and what is the smallest possible number of elements of R that are their own cubes? Explain.

1	2	3	4	total (40)	%

Prelim. course grade: %