

Name: _____

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Suppose G is a group such that every nontrivial element of G has order 2. Prove that G is abelian. Give an example of such a group that is not isomorphic to \mathbf{Z}_2 .
2. Prove that a group whose order is a prime must be cyclic.
3. Let $H = \{\alpha \in S_n : \alpha(1) = 1\}$ with $n \geq 5$. Prove that H is a subgroup of S_n . Prove or disprove that H a normal subgroup of S_5 .
4. Let H be as in the preceding problem. Suppose $\beta, \gamma \in S_n$ with $\beta(1) = \gamma(1)$. Prove that β and γ belong to the same left coset of H .
5. Suppose G is an abelian group whose order is odd. Prove that $\varphi : G \rightarrow G$ given by $\varphi(x) = x^2$ is an automorphism of G .

1	2	3	4	5	total (50)	%

Prelim. course grade: %