Real Analysis II, MAT 4223
Exam $\mathcal{N}^{\underline{\text{o}}}1$, 2/22/93
Instructor: D. Gokhman

Name: ______________________________

1. (20 pts.) Suppose $f : \mathbf{R} \to \mathbf{R}$ satisfies $|f(x)| \leq x^2$ for all $x \in \mathbf{R}$. Prove that f is differentiable at 0.

2. (20 pts.) True of false questions, circle your choice. If you choose TRUE, give a reason. If FALSE, provide a concrete counter example.

T F (a) If $f : \mathbf{R} \to \mathbf{R}$ is differentiable, then f' is continuous.

T F (b) If $f : \mathbf{R} \to \mathbf{R}$ is differentiable and $f'(x) \to 0$ as $x \to +\infty$, then there exists $c \in \mathbf{R}$ such that $f(x) \to c$ as $x \to +\infty$.

3. (20 pts.) Prove that if $f : \mathbf{R} \to \mathbf{R}$ is a differentiable function such that $f'(x) \in \mathbf{Q}$ for all $x \in \mathbf{R}$, then f' is constant.

4. (20 pts.) Prove that if $f : \mathbf{R} \to \mathbf{R}$ is a differentiable function such that f' is bounded, then f is uniformly continuous.

5. (20 pts.) Suppose $p(x)$ is a polynomial of degree $n \geq 15$.

   (a) What is the degree of p'?
   What is the degree of $p^{(k)}$, where $k \leq n$?

   (b) Find the limit of $p(x)e^{-x}$ as $x \to +\infty$.