University of Texas at San Antonio

Real Analysis II, MAT 4223
Final Exam, 5/4/92
Instructor: D. Gokhman

Name: ______________________________

1. (40 pts.) Suppose $f\colon [a,b] \to \mathbf{R}$ is continuous, $f(x) \geq 0$ for all $x \in [a,b]$ and
$$\int_a^b f(x)\,dx = 0.$$
Prove that $f(x) = 0$ for all $x \in [a,b]$.

2. (45 pts.) True of false questions, circle your choice. If you choose FALSE, give a reason. If TRUE, provide a concrete example.

   There exists a function $f\colon [0,1] \to \mathbf{R}$ such that

T F (a) f is bounded and not Riemann integrable.

T F (b) f is monotone and not Riemann integrable.

T F (c) f is Riemann integrable and has infinitely many discontinuities.

3. (35 pts.) Suppose $\{f_k\}$ is a sequence of bounded functions which converges uniformly to f. Prove that f is bounded.

4. (40 pts.) Suppose $\{f_k\}$ is a sequence of continuous functions which converges uniformly to f and x_k converges to x. Prove that $f_k(x_k)$ converges to $f(x)$.

5. (40 pts.) Suppose $f : \mathbf{R}^2 \to \mathbf{R}$ is a continuously differentiable function. A *level curve* of f is given by the equation $f(x,y) = k$, where k is a constant. Suppose $(x_0, y_0) \in \mathbf{R}^2$ is not a critical point of f.

   (a) Determine the slope of the line tangent to the level curve at (x_0, y_0).

   Hint: Apply the Implicit Function Theorem. You may have to take cases.

   (b) Show that the gradient of f at (x_0, y_0) is normal to the level curve.