Midterm 2 / 2019.3.22 / MAT 4213.002 / Real Analysis I

Name: _____

Please show all work. If you use a theorem, name it or state it.

1. Suppose $f : [0, \infty) \to \mathbf{R}$ is a decreasing function. Prove that f is continuous at 0 if and only if $f(0) = \sup \{f(x) : x > 0\}$.
2. Suppose $f(x) = x^3 \cos(1/x)$ for $x \neq 0$ and $f(0) = 0$. Prove that f is differentiable at 0.
3. Prove that for $t > 1$ we have $\ln(t) < t - 1$.

   Hint: Mean Value Theorem for the interval from 1 to t

1	2	3	total (30)