Midterm 1 / 2019.2.15 / MAT 4213.002 / Real Analysis I

Name: _____

Please show all work. If you use a theorem, name it or state it.

1. Show that if two continuous functions from reals to reals agree on rationals, they must be the same function.

2. Suppose $f : [0, 1] \to [0, 1]$ is continuous. Prove that f has a fixed point: $x \in [0, 1]$ such that $f(x) = x$.

3. Prove that the function $f(x) = \sqrt{x}$ is Lipschitz on the interval $[1, \infty)$. Why does it follow that f is uniformly continuous on $[0, \infty)$?

4. Give an example of a function $f : (0, 1) \to \mathbf{R}$ that is bounded, continuous, but not uniformly continuous. Explain.

1	2	3	4	total (40)