Real Analysis I / MAT 4213.001

Midterm 3 / November 25, 1998 / Instructor: D. Gokhman

Name: ______________________________

1. (10 pts.) Prove that the equation $\cos x = x$ has a real solution.

2. (25 pts.) Suppose $S \subseteq \mathbf{R}$ and $f\colon S \to \mathbf{R}$ is a function.
   Determine whether each of the following statements is true in general.
   If true, prove it. If false, give a specific counterexample.

   (a) If f is 1-1 and continuous, then f is monotone.

   (b) If f is 1-1 and continuous, then $f^{-1}\colon f(S) \to S$ is continuous.

   (c) If f is uniformly continuous and (x_n) is a Cauchy sequence in S, then $(f(x_n))$ is a Cauchy sequence.

   (d) If f is continuous and bounded, then f is uniformly continuous.

   (e) If $\forall x, t \in S \;\; |f(x) - f(t)| \le |x - t|$, then f is uniformly continuous.

3. (10 pts.) Suppose $f\colon \mathbf{R} \to \mathbf{R}$ is decreasing and $a \in \mathbf{R}$.
   Prove that $\lim\limits_{x \to a^+} f(x)$ exists.

1	2	3	total (45)	%