Real Analysis I / MAT 4213.001

Midterm 2 / November 4, 1998 / Instructor: D. Gokhman

Name: ______________________________

1. (20 pts.) Prove that the following functions $f : \mathbf{R} \setminus \{0\} \to \mathbf{R}$ have no limit in $[-\infty, \infty]$ at 0:

$$\text{(a) } f(x) = \frac{1}{x} \qquad \text{(b) } f(x) = \cos\left(\frac{1}{x}\right)$$

2. (20 pts.) Suppose $f : \mathbf{R} \to \mathbf{R}$ is defined by

$$f(x) = \begin{cases} x & \text{if } x \in \mathbf{Q}, \\ 0 & \text{if } x \in \mathbf{R} \setminus \mathbf{Q}. \end{cases}$$

Find the set $\{a \in \mathbf{R}\colon f \text{ is continuous at } a\}$. Prove your assertion.

3. (30 pts.) Suppose $S \subseteq \mathbf{R}$ and $f : S \to \mathbf{R}$ is a function.
Determine whether each of the following statements is true in general.
If true, prove it. If false, give a specific counterexample.

   (a) If $S = \mathbf{R}$, f is continuous, and A is an open subset of $\mathbf{R}$, then $f^{-1}(A)$ is an open subset of $\mathbf{R}$.

   (b) If $S = \mathbf{R}$, f is continuous and $A \subseteq \mathbf{R}$, then $\overline{f(A)} = f(\overline{A})$.

   (c) If $S = \mathbf{R}$, f is continuous, and A is a closed subset of $\mathbf{R}$, then $f(A)$ is a closed subset of $\mathbf{R}$.

   (d) If S is finite, then f is continuous.

   (e) If $S = (0, 1)$ and f is continuous, then $f(S)$ has a minimum.

1	2	3	total (70)	%