University of Texas at San Antonio

Real Analysis I, MAT 4213

Exam $\mathcal{N}^{\underline{o}}$2, 11/23/92

Instructor: D. Gokhman

Name: ____

1. (40 pts.)

   (a) Prove that if $\sum_{n=0}^{\infty} a_n$ is absolutely convergent, then $\sum_{n=0}^{\infty} a_n^2$ is convergent.

   (b) Provide an example that shows that the word 'absolutely' cannot be dropped from the hypothesis in part (a).

2. (30 pts.) Let a_n be a sequence such that $\forall r > 0\ \exists N\ \forall n > N\ 1 < a_n < n^r$ (an example of such a sequence is $a_n = \log n$). Let $\varepsilon > 0$. Determine (with proof) whether the following series converge

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a_n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n^{1+\varepsilon}}$

3. (30 pts.) Find the radius of convergence of the following power series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^{\sqrt{n}}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n x^n}{n!}$