University of Texas at San Antonio

Real Analysis I, MAT 4213
Exam $\mathcal{N}^{\underline{o}}1$, 10/17/91
Instructor: D. Gokhman

Name: ______________________________

1. (40 pts.) True of false questions, circle your choice. If you choose TRUE, give a sketch of proof of the assertion somewhere in the lower part of the page. If FALSE, provide a counterexample.

T F (a) If D_1 and D_2 are Dedekind cuts (rays in $\mathbf{Q}$), then $D_1 \cap D_2 \neq \emptyset$.

T F (b) If D is a Dedekind cut, then $D > 0 \Leftrightarrow 0 \notin D$.

T F (c) If $S \subseteq \mathbf{R}$, then S is finite $\Leftrightarrow \mathcal{L}(S) = \emptyset$, where $\mathcal{L}(S)$ denotes the set of all limit points of S.

T F (d) If S is an open subset of $\mathbf{R}$, then $\mathcal{L}(S) \not\subseteq S$.

2. (40 pts.) For each of the following subsets S of $\mathbf{R}$:

   (a) $S = \left\{r \in \mathbf{Q}\colon r^2 < 5\right\}$
   (b) $S = \left\{\frac{n}{n+1}\colon n \in \mathbf{N}\right\} \cup \left\{\frac{1}{n}\colon n \in \mathbf{N}\right\}$
   (c) $S = \bigcap_{n=1}^{\infty} \left(\frac{n-1}{n}, \frac{n-1}{n}\right)$
   (d) $S = \bigcup_{k=1}^{\infty} (-k, 5]$

   answer these questions:

   (i) Is S open, closed, neither or both?
   (ii) What is the closure $\overline{S}$?
   (iii) Do $\sup S$ and $\inf S$ exist, and if so, what are they?

3. (20 pts.) Suppose $\mathcal{F}$ is a collection of compact sets. Prove that $\bigcap_{F \in \mathcal{F}} F$ is compact.