Final exam / 2016.5.10 / MAT 3623.001 / Differential Equations II

Name: ____________________

Please show all work.

1. Solve the Clairaut equation $x = tx' + \frac{1}{x'}$.
2. Show that the system $x' = x - 4xy$, $y' = -2y + xy$ has a periodic solution.
3. Find the fundamental solution to the Airy equation $x'' = tx$ in power series form. Find the first 3 nonzero terms of each power series. Based on the form of the equation alone, what is your prediction for the radius of convergence of the power series?
4. Consider the dynamical system $x'(t) = -9x(t) + 8y(t), y'(t) = -12x(t) + 11y(t)$.
   (a) Show that the origin is the unique equilibrium of the system and determine its stability.
   (b) Find the invariant manifolds.
   (c) Sketch the invariant manifolds and a few trajectories of the system.
5. Solve the boundary value problem $x''(t) = 2 - 3t, x(0) = 0, x(1) - x'(1) = 0$.
6. Let $f(t) = 1 - t^2$. Obtain the first 3 nonzero terms of the Fourier expansion for f on the interval $[-1, 1]$. On the same graph sketch the function and the three partial sum approximations.
7. Solve the vibrating string equation $u_{tt} = c^2 u_{xx}$ for a string of length L with initial conditions $u(x, 0) = \sin \frac{5\pi x}{L}, u_t(x, 0) = 0$. On the same graph sketch $u(x, t)$ as functions of x for three different fixed values of t (starting with $t = 0$) to illustrate time evolution of the solution.
8. Find the steady state temperature of the disc $r \leq 3$, if the boundary $r = 3$ is held at $u(3, \theta) = 25 - 3\sin(2\theta)$ (in polar coordinates).

Fourier series: $f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L}]$,

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(t)\,dt, \quad a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}\,dt \ (n \geq 1), \quad b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi t}{L}\,dt$$

Laplacian: $\nabla^2 u = u_{xx} + u_{yy} = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}$

1	2	3	4	5	6	7	8	total (80)