Name: _____

Please show all work and justify your answers. If you use a theorem, name it or state it. Supply brief narration with your solutions and draw conclusions, including units as appropriate.

1. Use the definition to compute the Laplace transform of $t\,e^{-2t}\,u(t-3)$.
   For which s does the transform converge?

2. Find the inverse Laplace transform of $\ln(s-4)$.

   Hint: What operation in t-space corresponds to differentiation in s-space?

3. Use the method of Laplace transforms to solve the initial value problem

$$x'' + x = u(t-3), \qquad x(0) = 1, \quad x'(0) = 2$$

4. Find the Taylor series about $t = 0$ of $t^5(4+t^2)^{-1}$. Use the summation notation, but also write out the first three nonzero terms. What is the radius of convergence? Explain.

   Hint: Express the given function in terms of geometric series using algebra and substitution $x = -t^2/4$. Leave multiplication by t^5 for dessert.

5. Find the first three nonzero terms of the power series solution about $t = 0$ to the initial value problem

$$(t+1)x'' - x = 0, \qquad x(0) = 0, \quad x'(0) = 2$$

1	2	3	4	5	total (50)	%

Prelim. course grade: %