Differential Equations, mat 3613
Final, December 15, 1995
Instructor: D. Gokhman

Name: Pseudonym:

Show all work. Answers alone are not sufficient. Box the answers.

1. ( 80 pts .) Solve the following initial value problems and describe the behaviour of each solution for large $x$.
(a) $y^{\prime \prime}+4 y^{\prime}-5 y=0, y(0)=1, y^{\prime}(0)=0$
(b) $y^{\prime \prime}+4 y^{\prime}+5 y=0, y(0)=1, y^{\prime}(0)=0$
(c) $y^{\prime \prime}+4 y^{\prime}+4 y=0, y(-1)=2, y^{\prime}(-1)=1$
(d) $y^{\prime \prime \prime}+2 y^{\prime \prime}-5 y^{\prime}-6 y=0, y(0)=0, y^{\prime}(0)=0, y^{\prime \prime}(0)=1$
[Hint: Find one characteristic root by trial and error.]
2. ( 40 pts.) Find the general solution for each of the following equations:
(a) $y^{\prime \prime}+y=x(1+\sin x)$
(b) $y^{\prime \prime}+2 y^{\prime}+y=e^{-x} \log x$
3. (30 pts.) Find the terms up to and including $x^{5}$ of the power series solution for the initial value problem $(x-1) y^{\prime \prime}-x y^{\prime}+y=0, y(0)=-2, y^{\prime}(0)=6$
Extra credit: find the general form of the series and compute its radius of convergence.

| 1 a | 1 b | 1 c | 1 d | 2 a | 2 b | 3 | total (150) |
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