Engineering Analysis I, MAT 3253. Final Exam. 12/13/1993 Instructor: D. Gokhman Show all pertinent work, answers alone are not sufficient. Box the answers.

Name: \_\_\_\_\_

1. (40) Find the general solution of the homogeneous differential equation

$$tx^{2}dx + (t^{3} - x^{3}) dt = 0.$$

2. (40) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = \ln(t) \ e^{-t}.$$

3. (40) Find the general solution of the system

$$\frac{dx}{dt} - x + 2\frac{dy}{dt} = e^t, \qquad \frac{dx}{dt} - 5x + \frac{dy}{dt} = -2e^t.$$

4. (40) Express the following function in terms of unit step functions and calculate its Laplace transform.

$$f(t) = \begin{cases} 1, & t < 1\\ 0, & 1 \le t < 2\\ (t-2)^2, & 2 \le t \end{cases}$$

Extra credit (10): What happens if  $(t-2)^2$  is replaced by t?

5. (40) Find the solution of the initial value problem

$$\frac{dx}{dt} + 5x = \delta(t-2), \qquad x(0) = 10.$$

$\mathcal{L}\left\{t^n\right\} = n!/s^{n+1}$
$\mathcal{L}\left\{\delta(t)\right\} = 1$
$\mathcal{L}\left\{f'\right\} = s\mathcal{L}\left\{f\right\} - f(0)$

$$\Box \quad \mathcal{L} \{ e^{at} f \} = \mathcal{L} \{ f \} (s - a)$$
  
$$\Box \quad \mathcal{L} \{ f(t - a) \mathcal{U} (t - a) \} = e^{-as} \mathcal{L} \{ f \}$$

1	2	3	4	5	total $(200)$