Engineering Analysis I, mat 3253. Final Exam. 12/13/1993 Instructor: D. Gokhman Show all pertinent work, answers alone are not sufficient. Box the answers.

Name:

1. (40) Find the general solution of the homogeneous differential equation

$$
t x^{2} d x+\left(t^{3}-x^{3}\right) d t=0
$$

2. (40) Find the general solution of the differential equation

$$
\frac{d^{2} x}{d t^{2}}+2 \frac{d x}{d t}+x=\ln (t) e^{-t}
$$

3. (40) Find the general solution of the system

$$
\frac{d x}{d t}-x+2 \frac{d y}{d t}=e^{t}, \quad \frac{d x}{d t}-5 x+\frac{d y}{d t}=-2 e^{t}
$$

4. (40) Express the following function in terms of unit step functions and calculate its Laplace transform.

$$
f(t)= \begin{cases}1, & t<1 \\ 0, & 1 \leq t<2 \\ (t-2)^{2}, & 2 \leq t\end{cases}
$$

Extra credit (10): What happens if $(t-2)^{2}$ is replaced by $t$ ?
5. (40) Find the solution of the initial value problem

$$
\frac{d x}{d t}+5 x=\delta(t-2), \quad x(0)=10
$$

- $\mathcal{L}\left\{t^{n}\right\}=n!/ s^{n+1}$
$\square \mathcal{L}\left\{e^{a t} f\right\}=\mathcal{L}\{f\}(s-a)$
- $\mathcal{L}\{\delta(t)\}=1$
$\square \mathcal{L}\{f(t-a) \mathcal{U}(t-a)\}=e^{-a s} \mathcal{L}\{f\}$
$\square \mathcal{L}\left\{f^{\prime}\right\}=s \mathcal{L}\{f\}-f(0)$

| 1 | 2 | 3 | 4 | 5 | total (200) |
| :--- | :--- | :--- | :--- | :--- | :--- |
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