Name: $\qquad$
Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Use the method of Lagrange multipliers to find the global maximum and minimum of the scalar field $f(x, y)=2 x^{2}+3 y^{2}$ on the unit disc.
2. Compute the volume of the solid enclosed by the surfaces $z=1-x^{2}-y^{2}$ and $z=0$.
3. Find the scalar potential for the vector field $F=\left[3 x^{2}, z^{2} / y, 2 z \ln y\right]$ or show that such a potential doesn't exist.
4. Integrate $\omega=y d x-x d y$ around the unit circle counterclockwise. Compute the same integral using Green's theorem.
5. Compute the flux of $F=[2 x, 3 y, 0]$ through the surface $x^{2}+y^{2}=1,-1 \leq z \leq 1$ oriented with the normal away from the $z$ axis both directly and also using the divergence theorem.

| 1 | 2 | 3 | 4 | 5 | total (50) |
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