Midterm 1 / 2009.10.14 / Calculus for Applications / MAT 3243.001

Name: _____

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. In 2 dimensions sketch and label 3 nonempty level sets of $f(x,y) = \sqrt{4x^2 + y^2}$. Sketch the graph of $z = f(x,y)$ in 3 dimensions. Is f differentiable? Explain.

2. In each case determine whether the limit exists, and if so, find the limit.

$$\text{(a)} \lim_{[x,y]\to 0} \frac{xy - y^2}{x^2 + y^2} \qquad \text{(b)} \lim_{[x,y]\to 0} \frac{x^6 - x^2y^4}{x^2 + y^2}$$

3. The temperature distribution (in degrees Fahrenheit) at position $[x,y]$ (in miles) is given by $T(x,y) = 98 - x^2y$. You start walking from $[1,2]$ in the direction 30° south of east at 4 miles per hour. How fast is the temperature changing?

4. Let $f = e^{1+x+y^2}$. Compute the Hessian matrix for f and find the quadratic Taylor approximation to f at the origin.

5. A Petri dish 2 inches in diameter is used to grow a culture of *bacillus tularensis* and the population density is given by $d(x,y) = 2x^2 + y^2 - y + 3$ in millions of bacilli per square inch. Where is the population density the lowest? The highest?

   Hint: parametrize the boundary.

1	2	3	4	5	total (50)	%

Prelim. course grade: %