

Name: \_\_\_\_\_

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Sketch and label 5 level sets of  $f(x, y) = xy$ , including one at level 0.
2. In each case determine whether the limit exists, and if so, find the limit.

$$(a) \lim_{[x,y] \rightarrow 0} \frac{x^4 - y^4}{x^2 + y^2} \quad (b) \lim_{[x,y] \rightarrow 0} \frac{x^2 - y^2}{x^2 + y^2}$$

3. If a cucaracha crawls south at 1 cm/s, it notices an increase in temperature at the rate of  $2^\circ/\text{s}$ . If it crawls east at 1 cm/s, the temperature increases by  $4^\circ/\text{s}$ . What is the rate of change of temperature if the cucaracha crawls northeast at 2 cm/s?
4. Find the divergence and curl of  $[y^2z, \exp(xyz), x^2y]$ .
5. Let  $f = (1 + x^2 + y^2)^{-1}$ . Compute the Hessian matrix for  $f$  and find the quadratic Taylor approximation to  $f$  at the origin.

1	2	3	4	5	total (50)	%

Prelim. course grade: %