Name: $\qquad$
Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Use cylindrical coordinates to integrate $\left(x^{2}+y^{2}+3 z^{2}\right) d x d y d z$ over the solid $x^{2}+y^{2} \leq 4,-2 \leq z \leq 1$.
2. Either find a scalar potential for $F$ or explain why it fails to exist, where

$$
\text { (a) } F=[y,-x, 0] \quad \text { (b) } F=[x, y, z]
$$

3. Either find a vector potential for $F$ or explain why it fails to exist, where
(a) $F=[2 x,-y,-z]$
(b) $F=[3 x,-y,-z]$
4. Use Green's theorem to calculate the area under one arch of the cycloid $[x, y]=[t-\sin t, 1-\cos t]$ pictured below.


Hint: Find a 1-form $\omega$ such that $d \omega=d x d y$ and recall that $\int_{\Omega} d \omega=\int_{\partial \Omega} \omega$
5. Find the flux of $[7 x, 8 y, 9 z]$ through the unit sphere.

Hint: Don't do it directly.

| 1 | 2 | 3 | 4 | 5 | total (50) | $\%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |
| Prelim. course grade: |  |  |  |  |  | $\%$ |

