Final exam / 2007.12.6 / Calculus for Applications / MAT 3243.001

Name: ____

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Let $\mathbf{r} = [x, y, z]$ and $r = |\mathbf{r}|$. Express $\nabla\cdot(r^n\,\mathbf{r})$ in terms of r.
2. Let $\omega = x\,dx + y\,dy + z\,dz$ and $\eta = (x^2 + yz)\,dy\,dz + (y^2 + zx)\,dz\,dx + (z^2 + xy)\,dx\,dy$. Compute $d\eta$ and $\omega \wedge \eta$.
3. Given a steady temperature distribution $f(x, y) = x^y$, how quickly does the temperature change as you start moving from the point $[3, 2]$ towards $[2, 3]$ with speed 5?
4. Use cylindrical coordinates to parametrize the solid cone $z^2 = x^2 + y^2$, $-1 \le z \le 0$. Integrate $(x^2 + y^2 - z^2)\,dx\,dy\,dz$ over this cone.
5. Either find a scalar potential for $[3x^2, z^2/y, 2z \ln y]$ or explain why it fails to exist.
6. Either find a vector potential for $[xy^2z, -y^3z, x^2y + y^2z^2]$ or explain why it fails to exist.
7. Verify the fundamental theorem $\int_\Omega d\omega = \int_{\partial\Omega} \omega$ with $\omega = xz\,dx + yz\,dy + (x^2 + y^2)dz$ and the surface Ω given by $x^2 + y^2 + 2z = 1$, $z \ge 0$ oriented with the upward normal. Sketch.

   Hint: Parametrize Ω using cylindrical coordinates.

   Extra credit: Who first discovered the special case of the fundamental theorem that applies here?
8. Let F be a smooth vector field on $\mathbf{R}^3$ such that the flux of F through the lateral surface of a cone of volume b is q. If F has constant divergence c, what is the flux of F through the base of the cone? Explain.

1	2	3	4	5	6	7	8	total (80)	%