Name: $\qquad$
Please show all work and justify your statements. Label sketches, draw conclusions using complete sentences including units, and box your final answers as appropriate.

1. Wide Axle Company produces axles at two locations in quantities $q_{1}$ and $q_{2}$. The total cost of production is $2 q_{1}^{2}+q_{1} q_{2}+q_{2}^{2}+500$. Wide Axle wants to produce 200 axles. Use Lagrange multipliers to explain how the production should be split between the two locations in order to minimize cost?
2. Integrate $\eta=-y d x+x d y+z d z$ along the straight line segment from $[1,1,0]$ to $[3,2,1]$. Compute $d \eta$. Had we chosen a different path between the two points, would the integral remain the same? Explain.
3. Find an equation and a parametric formula for the plane tangent to the surface $[s t, s+t, s-t]$ at $[-2,1,3]$.
4. Compute the flux of $F=[x, y, z-1]$ through the unit disc in the $x-y$ plane. Then, use the divergence theorem to find the flux of $F$ through the top half of the unit sphere.
[Hint: the hemisphere and the disc together, with appropriate orientation, form the boundary of a solid.]
[Formulas: surface area of a sphere of radius $\rho$ is $4 \pi \rho^{2}$ and its volume is $\frac{4}{3} \pi \rho^{3}$ ]

| 1 | 2 | 3 | 4 | total (40) | $\%$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| Prelim. course grade: |  |  |  |  |  |

