Final exam / 2005.12.8 / Calculus for Applications / MAT 3243

Name: ______________________________

Please show all work and justify your statements. Label sketches, draw conclusions using complete sentences including units, and box your final answers as appropriate.

1. Compute the following linear approximations.
   (a) Find an equation for the plane tangent to the surface $\pi^y \ln z + x\cos(xz) = 0$ at the point $[\pi, 1, 1]$.
   (b) Find a parametric formula for the line tangent to $[\cos(\pi t), \sin(\pi t), 2t]$ at $[0, -1, 3]$.
2. Consider the vector field $F(x, y, z) = [x + yz, y + zx, z + xy]$.
   (a) Compute DF and $\nabla \cdot F$.
   (b) Find a scalar potential for F. What conclusion can you make about $\nabla \times F$? Explain.
3. Suppose $F = f(u, v, w)$, where $u = 2y + z$, $v = 3z + x$, $w = x + 2y$. Express the partial derivatives of F with respect to x, y, and z in terms of the partial derivatives of f with respect to u, v, and w.
4. For the scalar field $f(x, y) = x + 2y + \ln(xy)$ find all critical points and classify them using the Hessian criterion.
5. Consider the triangle formed by the coordinate axes in the plane and a line with intercepts a and b ($a, b > 0$). Use Lagrange multipliers to determine the dimensions of the rectangle with two sides along the coordinate axes, inscribed in the triangle, and having the largest possible area.
6. Integrate $\eta = y\,dx + x\,dy + z\,dz$ along the straight line segment from $[0, 1, 1]$ to $[1, 2, 3]$. Compute $d\eta$. Had we chosen a different path between the two points, would the integral remain the same? Explain.
7. Find an equation and a parametric formula for the plane tangent to the surface $[s^2t, st^2, st]$ at $[-2, 4, -2]$.
8. Compute the flux of $F = [x, y, 2 - z]$ through the unit disc in the x-y plane. Then, use the divergence theorem to find the flux of F through the top half of the unit sphere.

   [Hint: the hemisphere and the disc together, with appropriate orientation, form the boundary of a solid.]

   [Formulas: surface area of a sphere of radius ρ is $4\pi\rho^2$ and its volume is $\frac{4}{3}\pi\rho^3$]

1	2	3	4	5	6	7	8	total (80)