Calculus for Applications / MAT 3243.001

Midterm (extra) / October 22, 1997 / Instructor: D. Gokhman

Name: _____

Please show all work and box the final answers.

1. (10 pts.) Let $u = (-1, 2) \in \mathbf{R}^2$ and let $f\colon \mathbf{R}^2 \to \mathbf{R}$ be defined by $f(v) = \operatorname{comp}_u v$. In other words, f maps a vector to its component along u. Compute df. (Hint: let $v = (x, y)$ and express $f(v)$ as a function of x and y.)

2. (20 pts.) Consider the surface in $\mathbf{R}^3$ given by $(x^3 - 2z)(y + z^4)^5 = 1$. Find coefficients A, B, C, D such that $Ax + By + Cz = D$ gives the tangent plane to this surface at the point $(1, 1, 0)$.

3. (15 pts.) Parametrize the following curves. Specify the range for the parameter.

   (a) The straight line segment in $\mathbf{R}^3$ from $(-2, 1, 7)$ to $(6, 9, -1)$.

   (b) The circle of radius 2 in the y-z plane centered at $(0, 3, -2)$.

   (c) The graph of $y = e^x$ in the plane.

4. (20 pts.) Let $F\colon \mathbf{R}^3 \to \mathbf{R}^3$ be the vector field defined by $F(x, y, z) = \widehat{k} - y\,\widehat{\imath} + x\,\widehat{\jmath}$. Integrate F along the helical segment $\mathbf{c}(t) = 2\cos(t)\,\widehat{\imath} + 2\sin(t)\,\widehat{\jmath} + t\,\widehat{k}$, $-\pi \le t \le \pi$. What is the arclength of this segment?

5. (15 pts.) Let $F\colon \mathbf{R}^3 \to \mathbf{R}^3$ be defined by $F(x, y, z) = e^y\,\widehat{\imath} + (xe^y + ze^{yz})\,\widehat{\jmath} + ye^{yz}\,\widehat{k}$. Find $f\colon \mathbf{R}^3 \to \mathbf{R}$ such that $\operatorname{grad} f = F$ and $f(1, 0, 1) = 5$.

1	2	3	4	5	total (80)	%