Calculus for Applications, MAT 3243
Final, December 12, 1995
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Show all work. Answers alone are not sufficient.

1. (40 pts.) Let $\omega = e^x y^3 z\,dx + 3e^x y^2 z\,dy + e^x y^3\,dz$
   (a) Show that $d\omega = 0$ (i.e. $\operatorname{curl}(e^x y^3 z, 3e^x y^2 z, e^x y^3) = 0$).
   (b) Find $f(x,y,z)$ such that $df = \omega$ (i.e. $\operatorname{grad} f = (e^x y^3 z, 3e^x y^2 z, e^x y^3)$).
   [Hint: guess/check or integrate along a straight line $(0,0,0) \to (x,y,z)$]
   (c) Evaluate $\displaystyle\int_{(1,2,-2)}^{(0,-1,2)} \omega = \int_{(1,2,-2)}^{(0,-1,2)} e^x y^3 z\,dx + 3e^x y^2 z\,dy + e^x y^3\,dz$
   [Hint: you may use part (b) and the Fundamental Theorem of Calculus.]
   (d) What is the integral of ω around a closed curve? Explain.
2. (40 pts.) Let $F(x,y,z) = (3x, 3y, -7z)$. Find the flux $\displaystyle\int F \cdot dS$ through:
   (a) the cylinder $\Phi(\theta, z) = (2\cos\theta, 2\sin\theta, z)$, $0 \le \theta < 2\pi$, $-5 \le z \le 5$
   (b) the disc $\Phi(\rho, \theta) = (\rho\cos\theta, \rho\sin\theta, 5)$, $0 \le \rho \le 2$, $0 \le \theta < 2\pi$
3. (40 pts.) Evaluate the following integrals:
   (a) $\displaystyle\int_S 2xy^3 z\,dx + 3x^2 y^2 z\,dy + x^2 y^3\,dz$, where S is the unit circle in the x-y plane ($S = \{(x,y,z)\colon x^2 + y^2 = 1, z = 0\}$).
   (b) $\displaystyle\int_D e^y z\,dy\,dz - 5x^z\,dz\,dx + \log(xy)\,dx\,dy$, where D is the unit sphere ($D = \{(x,y,z)\colon x^2 + y^2 + z^2 = 1\}$).

   [Hint: S and D are boundaries.]

1a	1b	1c	1d	2a	2b	3a	3b	total (120)