Calculus for Applications, MAT 3243
Final, December 13, 1994
Instructor: D. Gokhman

Name: ______________________________

1. (20 pts.) INNER PRODUCT
   (a) Find two linearly independent vectors perpendicular to (1,1,1).
   (b) Find the angle between the diagonal of the unit cube and one of its sides.

2. (20 pts.) CHAIN RULE
   Let $F(x, y, z) = x + y^2, G(t, s) = (x(t, s), y(t, s), z(t, s)) = (t^2 - s^2, ts, t^2 + s^2)$.
   Find the derivative matrices of F, G and $F \circ G$.

3. (20 pts.) Find the quadratic function of x and y that approximates $f(x, y) = e^{x+y}$ at $(x_0, y_0) = (0, 0)$.

4. (20 pts.) CONSTRAINED EXTREMA
   (a) Sketch the unit circle and four different level curves of $f(x, y) = x + 2y$.
   (b) Find the absolute maximum and minimum of $f(x, y) = x + 2y$ on the unit circle.

5. (30 pts.) CURVE INTEGRAL
   (a) Find a parametric formula $\boldsymbol{r}(t) = (x(t), y(t))$ for the unit circle.
   (b) Sketch the unit circle. On the same graph sketch the vector field $\boldsymbol{F}(x, y) = (x, y)$ at four different points.
   (c) Integrate $\boldsymbol{F} \cdot d\boldsymbol{r}$ along the unit circle, where $\boldsymbol{F}$ and $\boldsymbol{r}$ are as in parts (a) and (b).

6. (20 pts.) SURFACE INTEGRAL
   (a) Find a parametric formula $\Phi(u, v) = (x(u, v), y(u, v), z(u, v))$ for the surface $x^2 + y^2 = 1$, $-1 \le z \le 1$. (Hint: you may let $u = \theta$ and $v = z$)
   (b) Let $\boldsymbol{F}(x, y, z) = (x, y, -y)$. Integrate $\boldsymbol{F} \cdot d\boldsymbol{S}$ through the surface of part (a).

Useful formulas: $\boldsymbol{u} \cdot \boldsymbol{v} = |\boldsymbol{u}|\,|\boldsymbol{v}| \cos\theta$ $\quad |\boldsymbol{u} \times \boldsymbol{v}| = |\boldsymbol{u}|\,|\boldsymbol{v}|\,|\sin\theta|$

Cylindrical coordinates: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ z \end{pmatrix}$ Spherical coordinates: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\sin\varphi\cos\theta \\ r\sin\varphi\sin\theta \\ r\cos\varphi \end{pmatrix}$

$$d\boldsymbol{r} = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \boldsymbol{r}'(t)\,dt, \qquad d\boldsymbol{S} = \begin{pmatrix} dy\,dz \\ dz\,dx \\ dx\,dy \end{pmatrix} = \left(\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}\right) du\,dv$$

1	2	3	4	5	6	total (130)