Midterm 2 / 2015.3.27 / MAT 3233.001 / Modern Algebra

Name: ____

Please show all work and justify your answers.

Assume throughout that R and S are commutative rings with unity.

1. Find all integer solutions x of the linear congruence equation $6x \equiv 16 \bmod 28$.

2. Show that multiplicative inverses in a ring, when they exist, are unique by proving that if $a, a', a'' \in R$ with $aa' = 1$ and $aa'' = 1$, then $a' = a''$.

3. Suppose $\varphi\colon R \to S$ a ring homomorphism. Show directly from the definition that for any unit $x \in R$ we have $\varphi(x^{-1}) = \varphi(x)^{-1}$.

4. Suppose R is a ring, $a \in R$ and $\varphi\colon R \to R$ is an additive group homomorphism defined by $\varphi(x) = ax$. Show that φ is one-to-one if and only if a is neither zero nor a zero divisor.

1	2	3	4	total (40)