Midterm 1 / 2015.2.20 / MAT 3233.001 / Modern Algebra

Name: $\qquad$
Please show all work and justify your answers.

1. Find all natural numbers $n$ such that $2 n!<n^{n}$. Prove your conclusion by induction.
2. Use the Extended Euclid's Algorithm to find $\operatorname{gcd}(324,148)$ and $s, t \in \mathbf{Z}$ such that $\operatorname{gcd}(324,148)=324 s+148 t$. Show steps.
3. Prove that $\operatorname{lcm}(a, b)$ divides any common multiple of $a$ and $b$.
4. Prove that $5^{\frac{1}{3}}$ is irrational.

| 1 | 2 | 3 | 4 | total (40) |
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