

Name: \_\_\_\_\_

Please show all work.

1. Partition  $U_{19}$  into cosets of  $\langle 12 \rangle$ .
2. If  $G$  is a commutative group and  $H < G$ , prove that  $x, y \in G$  give the same coset of  $H$  (i.e.  $xH = yH$ ) exactly when  $xy^{-1} \in H$ .
3. If  $[x]_m \in \mathbf{Z}_m$ , show that  $\langle x \rangle = \mathbf{Z}_m$  exactly when  $[1]_m \in \langle x \rangle$ . Then show that this happens exactly when  $x$  is coprime to  $m$ .

[Hint: Bezout]

4. Find the solution set for the system of congruences

$$8x \equiv 2 \pmod{18}$$

$$9x \equiv 28 \pmod{30}$$

5. Exhibit an injective group homomorphism from  $U_7$  to  $S_7$ .

1	2	3	4	5	total (50)	%

Prelim. course grade: %