Name: $\qquad$
Please show all work.

1. (a) Find a generator for $U_{7}$. (b) Show that $U_{8}$ is not cyclic.
2. Partition $U_{13}$ into cosets of the subgroup generated by 3 .
3. Let $f: \mathbf{Z}_{15} \rightarrow \mathbf{Z}_{15}$ be the abelian group homomorphism defined by $f(a)=5 a$. What are the kernel and the image of $f$ ? Are they subgroups of $\mathbf{Z}_{15}$ ?
4. Solve the system of congruences $x \equiv 1 \bmod 2, x \equiv 5 \bmod 8, x \equiv 3 \bmod 5$.
5. Find the gcd of $x^{3}-1$ and $x^{2}-1$. What is the corresponding Bezout relation?

| 1 | 2 | 3 | 4 | 5 | total (50) | $\%$ |
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| Prelim. course grade: $\%$ |  |  |  |  |  |  |

