Final / May 13, 2005 / MAT 3233.001 / Modern Algebra

Name: ____

Please show all work.

1. Prove by induction that $n! \leq n^n$ for all $n \geq 1$.
2. Compute 5^{23} modulo 7 by repeated squaring and reduction. Show work.
3. Solve the linear congruence $20x \equiv 5 \bmod 23$.
4. Solve the system of congruences $x \equiv 3 \bmod 4$, $x \equiv 4 \bmod 5$, $x \equiv 5 \bmod 6$.
5. Find the gcd of $x^3 + 1$ and $x^5 + 1$ in $\mathbf{Q}[x]$. What is the corresponding Bezout relation?
6. If a, b, q, r are natural numbers such that $b = aq + r$, prove that $\gcd(a, b) = \gcd(a, r)$.
7. Prove that any nonzero element in a finite commutative ring is either a unit or a zero divisor.
8. Prove that U_4 is cyclic, but the symmetric group S_3 is not.
9. Construct an injective group homomorphism from U_4 to S_3. Verify that the function you have constructed is indeed a group homomorphism.
10. Let G be the abelian group $\mathbf{Z} \times \mathbf{Z}$ (the integer lattice in the plane). Sketch the subgroup of G generated by (1,2). Sketch one nontrivial coset of this subgroup.

1	2	3	4	5	6	7	8	9	10	total (100)