Name: $\qquad$
Please show all work and explain your answers.

1. Let $f(z)=(\operatorname{Re} z)^{2}+(\operatorname{Im} z)^{2}-2 i \operatorname{Re} z \operatorname{Im} z$. At which $z$ is $f(z)$ complex differentiable? Analytic? Explain.
2. Integrate $\operatorname{Im} z d z$ along the straight line segment from the origin to $3 i-4$.
3. Integrate $\frac{e^{z}}{2 z^{2}+z} d z$ counterclockwise around the unit circle.
4. Expand $1 / z$ in a Taylor series at $z=3 i-4$. What is the disc of convergence?
5. Expand $\left(z^{2}-3 i z-2\right)^{-1}$ in a Laurent series centered at the origin and valid in the annulus $\{z: 1<|z|<2\}$.
6. Integrate $\sin (\csc z)$ around the unit circle.
7. Use Rouché's theorem to determine the number of zeros, counted with multiplicity, of $z^{3}-8 z^{2}+11 z-1$ in the annulus $\{z: 1<|z|<2\}$.
8. Find a fractional linear transformation that maps the unit disc to the exterior of the circle of radius 2 centered at $i$.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | total (80) |
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