Final / May 12, 2005 / MAT 3223.001 / Functions of a complex variable

Name: _____

Please show all work and explain your answers. Cite any theorems you use in your reasoning.

1. Find and sketch the cube roots of $1 - i$.
2. Describe the geometric effect of plane transformations $f(z) = 2iz + 1$ and $g(z) = \overline{z}$.
3. Determine where $|z|^3$ is complex differentiable.
4. Integrate $\dfrac{e^z\,dz}{z(2z+1)}$ around the unit circle.
5. Integrate $\dfrac{\sin(z^2)\,dz}{z^3}$ around the unit circle.
6. Integrate $\Re[z]\,\Im[z]\;dz$ along the straight line segment from 0 to $1 + i$.
7. Prove that a nonconstant entire function must have some purely imaginary values.
8. Find the first 3 nonzero terms of the Taylor expansion of $\dfrac{z^7}{\tan z}$ at $z = 0$.
9. Expand $\dfrac{1}{z^2+1}$ in a Laurent series convergent in a punctured disc centered at i.
   What is the open annulus of convergence? Is the convergence uniform on the annulus?
10. Find all singularities of $z \tan \dfrac{1}{z}$. Are any of them removable?

1	2	3	4	5	6	7	8	9	10	total (100)