Complex Variables / MAT 3223.001
Final exam / 2002.5.9 / Instructor: D. Gokhman

Name: ______________________________

1. (15 pts.) Sketch and describe the locus in $\mathbf{C}$ of each of the following equations.

   (a) $|z+i| = |z-1|$ (b) $\operatorname{Arg}(z+2i) = -\dfrac{4\pi}{3}$ (c) $\operatorname{Im}(z^2) = 1$

2. (20 pts.) Find the power series expansion of each of the following functions at the given center c. Write out the first 3 nonzero terms. Sketch and label all singularities and branch points of the function, if any. Sketch the disc of convergence of the power series. What is the radius of convergence?

   (a) $\dfrac{1}{3+z}, c=0$ (b) $\dfrac{1}{1-z}, c=2i$ (c) $\log(z), c=2i$ (d) $\dfrac{z^7}{e^{iz^2}}, c=0$

3. (15 pts.) Consider $f\colon \mathbf{C} \to \mathbf{C}$.

   (a) Suppose $f(z) = 2i\log(z)$. Describe geometrically the local behaviour of f in the vicinity of $z = 1+i$.
   (b) Suppose $f(z) = 1/z^2$. Near which z is the local effect of f a dilation by a factor 2 and a clockwise right angle rotation?
   (c) Suppose the global effect of f is a dilation by a factor of 1/3 composed with a clockwise rotation by $\pi/3$. Write down a possible formula for $f(z)$.

4. (10 pts.) Find the set of all points in the complex plane, where each of the following functions $f\colon \mathbf{C} \to \mathbf{C}$ is complex differentiable. Sketch this set.

   In each case, for those z where f is complex differentiable, find $f'(z)$.

   (a) $f(x+iy) = x^2 + iy^2$ (b) $f(x+iy) = x^3 + y^3 - 3ixy$

5. (10 pts.) Show that 1 root of $z^4 + z^2 - 5z - 1$ is in the unit disk, while the other 3 are in the annulus $\{z\colon 1 \le z \le 2\}$.

6. (20 pts.) Evaluate the following integrals along the given paths (sketch):

   (a) $\displaystyle\int_\gamma \overline{z}\,dz$, where γ is the straight line segment from i to 1
   (b) $\displaystyle\int_\gamma \overline{z}\,dz$, where $\gamma = \{z\colon\ |z+1-i| = 1\}$
   (c) $\displaystyle\int_\gamma \frac{z\,dz}{z^2 - i}$, where $\gamma = \{z\colon\ |z+1-i| = 1\}$
   (d) $\displaystyle\int_\gamma \frac{dz}{z(\cos z - 1)}$, where γ is the unit circle

7. (10 pts.) Let $I(r) = \displaystyle\int_\gamma \frac{z-i}{z^4+1}\,dz$, where $\gamma = \{re^{it}\colon \pi \le t \le 2\pi\}$ with $r > 1$.

   Estimate $|I(r)|$ and show that $I(r) \to 0$ as $r \to \infty$.

1	2	3	4	5	6	7	total (100)	%