Complex Variables / MAT 3223.001

Midterm 1 / February 23, 1998 / Instructor: D. Gokhman

Name: _____

Please show all work and box the answers, where appropriate.

1. (10 pts.) Sketch the following subsets of the complex plane.

   (a) $\{z\colon\ |z+i| = |z-1-i|\}$ (b) $\{z\colon\ |z+i| = 2\}$

2. (10 pts.) Let $f(z) = \dfrac{z+i}{z-i}$. Find and sketch:

   (a) $f^{-1}(\{w\colon\ |w| = 1\})$ (b) $f(\{z\colon \operatorname{Re} z = 0\})$

3. (20 pts.)

   (a) Show that if z is a solution of $z^n = a$ and z_0 is a solution of $z^n = 1$, then $z_0 z$ is a solution of $z^n = a$.

   (b) Find all solutions of $z^3 = 1$.

   (c) Find a solution of $z^3 = 8i$. (Hint: Express $8i$ in polar form.)

   (d) Find all solutions of $z^3 = 8i$ and express them in polar and cartesian form. (Hint: You may use (a–c).)

   (e) Check your answers to part (d).

4. (20 pts.) Find the Maclaurin series expansion of each of the following functions and determine its radius of convergence.

   (a) $\dfrac{1}{1+2z}$ (b) $\dfrac{1}{i+z}$

5. (14 pts.) Mix'n'match.

____	(a) $f(z) = z$	(i) translation
____	(b) $f(z) = -z$	(ii) isotropic expansion/contraction
____	(c) $f(z) = \overline{z}$	(iii) rotation
____	(d) $f(z) = -\overline{z}$	(iv) identity
____	(e) $f(z) = e^{i\theta} z$	(v) reflection with respect to the x axis
____	(f) $f(z) = a + z$	(vi) reflection with respect to the y axis
____	(g) $f(z) = rz$	(vii) reflection with respect to the origin

1	2	3	4	5	total (74)	%