Complex Variables, MAT 3223
Final, May 11,1994
Instructor: D. Gokhman

Name: ______________________________

1. (40 pts.) Express the following quantities in terms of $x = \operatorname{Re} z$, $y = \operatorname{Im} z$, $r = |z|$, $\theta = \arg z$:

   (a) $|e^z|$ (b) $\arg e^z$ (c) $\operatorname{Re} \log z$ (d) $\operatorname{Im} \log z$

2. (30 pts.) For each of the following functions $f(z)$ find the largest subset of the complex plane, where $f(z)$ is differentiable.

   (a) $f(z) = 1/(z^4 - 8iz)$ (b) $f(z) = |z|^2$ (c) $f(z) = \dfrac{|z|^2}{\overline{z}}$

3. (40 pts.) Mix'n'match.

__ (a) $f(z) = z$	(i) translation
__ (b) $f(z) = -z$	(ii) scale
__ (c) $f(z) = \overline{z}$	(iii) rotation
__ (d) $f(z) = -\overline{z}$	(iv) identity
__ (e) $f(z) = 1/\overline{z}$	(v) reflection with respect to the x axis
__ (f) $f(z) = e^{\vec{i}\,\theta} z$	(vi) reflection with respect to the y axis
__ (g) $f(z) = z_0 + z$	(vii) reflection with respect to the origin
__ (h) $f(z) = rz$	(viii) inversion with respect to the unit circle

4. (30 pts.) Sketch the sector $\{z \in \mathbf{C}\colon\ |z| \leq 1, \pi/4 \leq \arg z \leq \pi/2\}$ and its images under the mappings defined in 3c, 3e, 3f with $\theta = \pi/2$, and $f(z) = \vec{i}\ /z$.

5. (80 pts.) Integrate the following functions along the indicated contours. Assume that closed contours are traversed once in the positive direction.

   (a) $g(z) = 1/z$ along the top half of the unit circle counterclockwise

   (b) $g(z) = \operatorname{Im} z$ along the line segment from 0 to $1+\ \vec{i}$

   (c) $g(z) = 1/(z^3 + 4z)$ around the circle of radius 2 centered at $-3\ \vec{i}$

   (d) $g(z) = 1/(z^3 + 2z^2)$ around the unit circle

1	2	3	4	5	6	7	8	total (330)

THE UNIVERSITY OF TEXAS AT SAN ANTONIO

6. (20 pts.) Find the Taylor series for $\log z$ at 1. What is the radius of convergence?

7. (60 pts.) Find the Laurent series in powers of z for the following functions with the indicated regions of validity:

   (a) $ze^{1/z}$ for all $z \neq 0$

   (b) $1/(1+z)$ for $|z| > 1$

   (c) $1/[(1+z)(2+z)]$ for $1 < |z| < 2$

8. (30 pts.) True / false.

   Assume that $f(z)$ is differentiable on a connected open set $U \subseteq \mathbf{C}$.

T F (a) $f(z)$ has a second derivative on U.

T F (b) The integral of $f(z)$ along any closed contour in U is 0.

T F (c) If $0 \in U$, the Maclaurin series of $f(z)$ converges to $f(z)$ on an open disk centered at 0.

T F (d) If $|f(z)|$ is constant on U, then so is $f(z)$.

T F (e) If $|f(z)|$ attains its maximum at $z \in U$, then $f(z)$ is constant on U.

T F (f) If $f(z)$ is not constant, then there exists z such that $f(z) = 0$.