

Name: \_\_\_\_\_

Please show all work. If you use a theorem, name it or state it.

1. Suppose  $A, B$  are nonempty bounded subsets of  $\mathbf{R}$ . Let  $A + B = \{a + b: a \in A, b \in B\}$ . Prove that  $\inf(A + B) = \inf A + \inf B$ .
2. Prove that the sequence  $(-1)^n \frac{n}{n+1}$  diverges.
3. Suppose  $A \neq \emptyset$  and bounded below. Prove there is a sequence  $(a_n)$  in  $A$  such that  $a_n \rightarrow \inf A$ .
4. Suppose  $x_1 = 1$  and  $x_n = \sqrt{x_{n-1} + 2}$  for  $n > 1$ . Show that the sequence  $(x_n)$  is monotone increasing and bounded above, thus convergent. Find the limit.

1	2	3	4	total (40)