Name: _____

Please show all work. If you use a theorem, name it or state it.

1. Let $c \in \mathbf{Q}$ and $C = \{r \in \mathbf{Q} \colon r > c\}$.

   (a) Prove that C is a Dedekind cut (C represents the real number c).

   (b) Suppose D is a Dedekind cut. Prove that $D < C$ if and only if $c \in D$.

   Hint: $D < C \Leftrightarrow C$ is a proper subset of D.

2. Find all real x such that $4 < |x + 2| + |x - 1| < 5$.

3. For each of sup/inf/max/min either find it or state it doesn't exist for the set $\{1/n^2 \colon n \in \mathbf{N}\}$. Prove your assertions.

4. Suppose A, B are nonempty bounded subsets of $\mathbf{R}$. Prove that $\sup(A \cup B) = \max\{\sup A, \sup B\}$.

5. Does the sequence $\dfrac{n}{n+1}$ converge? Prove your assertion. Same for the sequence $(-1)^n \dfrac{n}{n+1}$.

1	2	3	4	5	total (50)